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Evaluation of Plastic Deformation During Metal Forming by Using Lode Parameter

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In the present research, the relationship between the strain increment and Lode parameter was obtained by using Lode parameter and Levy-Mises equation. The quantitative relation between different strain increments was presented and the relationship between plastic deformation types and Lode parameter was demonstrated. The relations between plastic deformation types and stress states were revealed and they were applied into typical metal forming process. The strain types in the workpiece were analyzed based on the numerical visualization of the values of Lode parameter. This research reveals the important role of the Lode parameter in the analysis of metal flow behavior during metal forming process.

Keywords deformation type, Lode parameter, plastic deformation, strain increment

1. Introduction

There are three types of plastic deformation due to the constant volume principle: elongation deformation, plane strain deformation, and compression deformation (Ref 1, 2). The type of deformation is determined by the stress state, which is still a basic problem in engineering plasticity.

The stress state of a point under consideration can be described by six independent stress components in a mutually perpendicular coordinate system. The value and orientation of the components of stress tensor will depend on coordinate system, but the Lode parameter and the stress invariants are independent of the coordinate system. Therefore, the eigenvalues of stress tensor play an important role in the description of plastic deformation (Ref 3-5).

The value of the intermediate principal stress can be evaluated by Lode parameter, which also has a certain relationship with the strain type. So Lode parameter can be used to discriminate different strain types and the complexity of deformation (Ref 6). Therefore, by analyzing the value of Lode parameter, some improvements can be made to change the stress state, update deformation condition, and finally enhance metal forming property (Ref 7, 8).

Based on the relationship between the stress condition and the Lode parameter, the strain increment ratio and plastic deformation type were determined (Ref 9). The relationship between Lode parameter and the plastic deformation type was established quantitatively, and the corresponding relation between the plastic deformation type and the stress condition was proposed (Ref 2). By using the Lode parameter cloud chart obtained from the post-processing of a FE code, we can know the strain type within the workpiece, which must be significantly helpful for analyzing metal flow behavior, defects generation, and the effects of processing parameters.

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2. Quantitative Relation Between Lode Parameter and the Second Deviatoric Stress Invariant J_2

The second deviatoric stress invariant is shown in Eq 1:

$$J_2 = -\left(\sigma_1'\sigma_2' + \sigma_1'\sigma_3' + \sigma_2'\sigma_3'\right) \quad \text{(Eq 1)}$$

where $\sigma_1', \sigma_2', \sigma_3'$ are components of the deviatoric stress tensor. Since

$$\sigma_1' + \sigma_2' + \sigma_3' = 0 \quad \text{(Eq 2)}$$

And the shapes of the Mohr circles of deviatoric stress and stress tensor are the same,

$$\mu_\sigma = \frac{\sigma_2' - \frac{\sigma_1'+\sigma_3'}{2}}{\frac{\sigma_1'-\sigma_3'}{2}} \quad \text{(Eq 3)}$$

Nomenclature	
$\sigma_1, \sigma_2, \sigma_3$	1st, 2nd, and 3rd principal stresses
$\sigma_1', \sigma_2', \sigma_3'$	1st, 2nd, and 3rd principal deviatoric stresses
σ_m	mean stress
σ_S	equivalent stress
J_2	the second deviatoric stress invariant
μ_σ	Lode parameter of stress
$d\varepsilon_1, d\varepsilon_2, d\varepsilon_3$	1st, 2nd, and 3rd different strain increments
R	stress coefficient
β	coefficient without dimension
$d\lambda$	instantaneous constant
λ	ratio of height to diameter
X_1, X_2, X_3	coefficients relation between strain increments

Eq 4 can be obtained from Eq 2 and 3:

$$\mu_\sigma \frac{\sigma_1' - \sigma_3'}{2} = \frac{3}{2}\sigma_2' \quad \text{(Eq 4)}$$

According to von Mises yield criterion:

$$\sigma_1' - \sigma_3' = \frac{3\sigma_S}{\sqrt{3+\mu_\sigma^2}} \quad \text{(Eq 5)}$$

From Eq 2, 4, and 5, Eq 6 was obtained:

$$\begin{cases} \sigma_1' = \frac{(3-\mu_\sigma)R}{3} \\ \sigma_2' = \frac{2}{3}\mu_\sigma R \\ \sigma_3' = -\frac{(3+\mu_\sigma)R}{3} \end{cases} \quad \text{(Eq 6)}$$

where

$$R = \frac{\sigma_S}{\sqrt{3+\mu_\sigma^2}} \quad \text{(Eq 7)}$$

The equivalent stress can be written as:

$$\sigma_S = \sqrt{\frac{3}{2}\left[(\sigma_1-\sigma_m)^2 + (\sigma_2-\sigma_m)^2 + (\sigma_3-\sigma_m)^2\right]} \quad \text{(Eq 8)}$$

$$\frac{1}{3}\sigma_S^2 = \frac{1}{2}\left[\sigma_1'^2 + \sigma_2'^2 + \sigma_3'^2\right] \quad \text{(Eq 9)}$$

Eq 10 can be obtained from Eq 1 and 9:

$$J_2 = \frac{-\left(\sigma_1'\sigma_2' + \sigma_1'\sigma_3' + \sigma_2'\sigma_3'\right)}{\frac{1}{2}\left[\sigma_1'^2 + \sigma_2'^2 + \sigma_3'^2\right]} \times \frac{1}{3}\sigma_S^2 \quad \text{(Eq 10)}$$

and

$$\beta = \frac{-\left(\sigma_1'\sigma_2' + \sigma_1'\sigma_3' + \sigma_2'\sigma_3'\right)}{\frac{1}{2}\left[\sigma_1'^2 + \sigma_2'^2 + \sigma_3'^2\right]} \quad \text{(Eq 11)}$$

Eq 11 was substituted to β and simplified to Eq 12:

$$\beta = \frac{3\left(\mu_\sigma^2 + 3\right)}{(3-\mu_\sigma)^2} \quad \text{(Eq 12)}$$

For a certain stress state, $-1 \le \mu_\sigma \le 1$, the value of β can be obtained and the relationship is shown in Fig. 1.

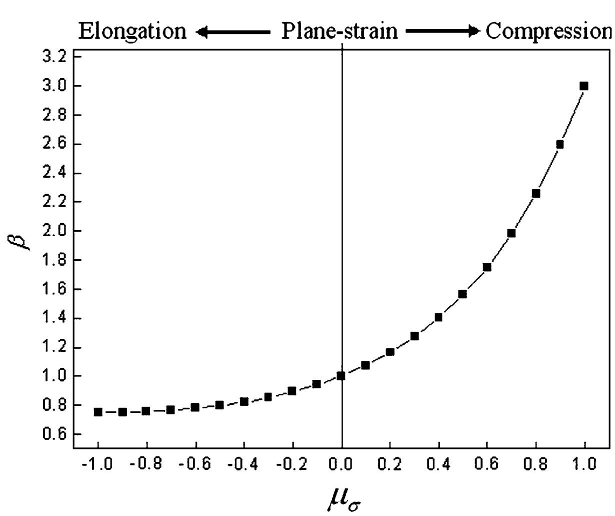

Fig. 1 The relationship between Lode parameter μ_σ and value of β

As can be seen from the figure, when the materials are in the plastic deformation state, there is some quantitative relationship between the Lode parameter μ_σ and the second deviatoric stress invariant J_2. The numerical value of J_2 increases with increasing Lode parameter μ_σ, and the degree of deformation of the work-hardening material increases. Therefore, Lode parameter μ_σ can be considered as one of the most important criteria for evaluating the degree of deformation of materials.

3. Quantitative Equation of Strain Incremental Ratio

Lode parameter $\mu_\sigma = \frac{2\sigma_2-\sigma_1-\sigma_3}{\sigma_1-\sigma_3}$ varies between -1 and 1, i.e. $-1 \le \mu_\sigma \le 1$, and the value of μ_σ is independent of any extra hydrostatic stress (Ref 10).

According to the Levy-Mise equations, the principal stress can be expressed as:

$$\frac{d\varepsilon_1}{\sigma_1-\sigma_m} = \frac{d\varepsilon_2}{\sigma_2-\sigma_m} = \frac{d\varepsilon_3}{\sigma_3-\sigma_m} = d\lambda > 0 \quad \text{(Eq 13)}$$

where $d\lambda$ is an instantaneous constant, which is related to the material properties. The denominator of Eq 13 is stress deviator component, and the member is the corresponding strain increment. The equation indicates a proportional relation between the strain increment and the corresponding stress deviator component, the Mohr strain increment circle is similar to that of stress deviator component, as shown in Fig. 2.

Similar Mohr circles and relations between stress and strain increment can be obtained when $\mu_\sigma = 0$ and $\mu_\sigma < 0$. Therefore, (1) the value of μ_σ indicates the strain type, i.e., (1) compression strain happens when $\mu_\sigma > 0$, elongation strain occurs when $\mu_\sigma < 0$ while it will be the plain strain situation when $\mu_\sigma = 0$; (2) the value of stress will be changed if an extra hydrostatic pressure is superimposed, but μ_σ will not be altered, therefore the strain type is not changed; and (3) the symbol of deviator stress component is the same with that of the strain increment (Ref 11).

Since the Mohr circles of stress and the strain increment are similar, thus,

$$\frac{d\varepsilon_3}{d\varepsilon_1} = \frac{\mu_\sigma - 3}{\mu_\sigma + 3} \quad \text{(Eq 14)}$$

According to the constant volume principle,

$$d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0 \quad \text{(Eq 15)}$$

Eq 16 can be obtained by substituting Eq 15 into Eq 14 and divided by $d\varepsilon_1$:

$$\frac{d\varepsilon_2}{d\varepsilon_1} = -1 - \frac{\mu_\sigma - 3}{\mu_\sigma + 3} \quad \text{(Eq 16)}$$

It can be seen from Eq 14 and 16 that, although the values of $d\varepsilon_1, d\varepsilon_2, d\varepsilon_3$ cannot be obtained, μ_σ can be utilized to indicate the proportional relation between different strain increments quantitatively.

According to the consistent relationship between the stress and strain, when $d\varepsilon_1 > 0$ and $d\varepsilon_3 < 0$, the strain type is determined by the symbol of $d\varepsilon_2$. Therefore, $d\varepsilon_1$ can be

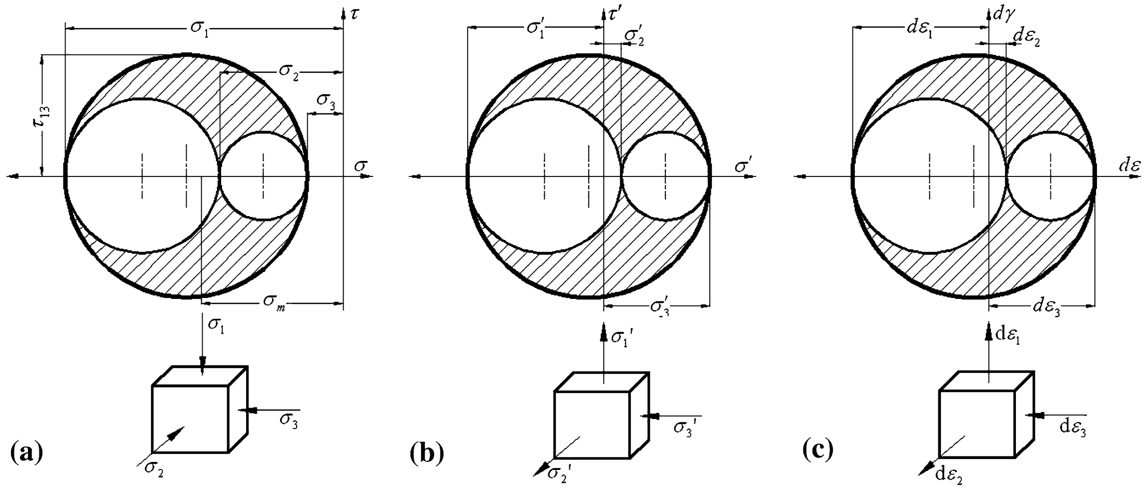

Fig. 2 Schematic diagram of Mohr circle form when $\mu_\sigma > 0$: (a) Mohr stress circle, (b) Mohr stress component circle, and (c) Mohr strain increment circle

considered as a constant coefficient and different strain increment values can be obtained:

$$d\varepsilon_1 = d\varepsilon_1,\ d\varepsilon_2 = \frac{-2\mu_\sigma}{\mu_\sigma + 3} d\varepsilon_1,\ d\varepsilon_3 = \frac{\mu_\sigma - 3}{\mu_\sigma + 3} d\varepsilon_1, \quad d\varepsilon_1 > 0 \qquad \text{(Eq 17)}$$

Let $d\varepsilon_1 = X_1 d\varepsilon_1$, $d\varepsilon_2 = X_2 d\varepsilon_1$, and $d\varepsilon_3 = X_3 d\varepsilon_1$, where X_1, X_2, X_3 are coefficients.

Thus:

$$X_1 = 1, \quad X_2 = \frac{-2\mu_\sigma}{\mu_\sigma + 3}, \quad X_3 = \frac{\mu_\sigma - 3}{\mu_\sigma + 3} \qquad \text{(Eq 18)}$$

Since the value of Lode parameter varies between −1 and 1, the value ranges of $d\varepsilon_1, d\varepsilon_2, d\varepsilon_3$ can be determined. For a certain stress state, namely, when the Lode parameter μ_σ is a constant, the values of X_2, X_3 can be obtained by using Eq 18, the values of the strain increments on the three principal directions and the quantitative relation between X_1, X_2, X_3 and μ_σ can be also determined as can be seen in Fig. 3. In the above-mentioned transformation procedures, $d\varepsilon_1$ was considered as a constant and its coefficient $X_1 = 1$, therefore, we can also consider Eq 18 as the relation between Lode parameter and the other two constants X_2, X_3.

The macroscale metal flow behavior at a point under consideration can be obtained from the quantitative relationship of Lode parameter and the strain increments in three principal directions, which will significantly simplify engineering plasticity problems.

Figure 4 shows three kinds of typical stress conditions. The value of Lode parameter and relative strain increments were calculated from the proposed equations.

It can be seen that under a certain stress condition, not only can we obtain the different deformation types, i.e. compression, elongation, and plane-strain deformation, but also we can describe the relationship between strain increment and Lode parameter quantitatively.

Lode parameter μ_σ can be used as a criterion of the strain type; in addition, it can be used to determine the strain increments in three principal directions and their relative relationship. Therefore, Lode parameter μ_σ can be used to quantitatively demonstrate the metal flow behavior under a certain stress condition.

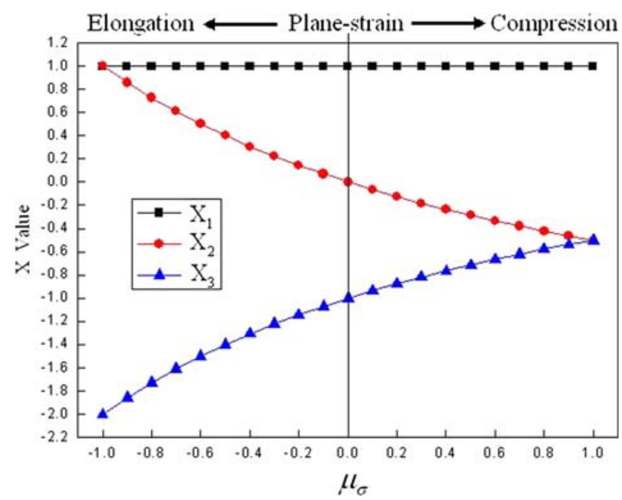

Fig. 3 The changing relation between Lode parameter and various coefficients

4. Applications

Based on the relationship between Lode parameter and the strain type, the values of Lode parameter were calculated and visualized and the strain types within the workpiece were analyzed for two typical metal forming process, i.e. ring compression and the extrusion process.

4.1 Ring Compression

Figure 4 shows the division of deformation area for the ring compression process with different height-diameter ratios. The shadow part is the plastic area.

As can be seen from Fig. 5(a), when the height-diameter ratio is 0.5, the zero value line of Lode parameter locates close to

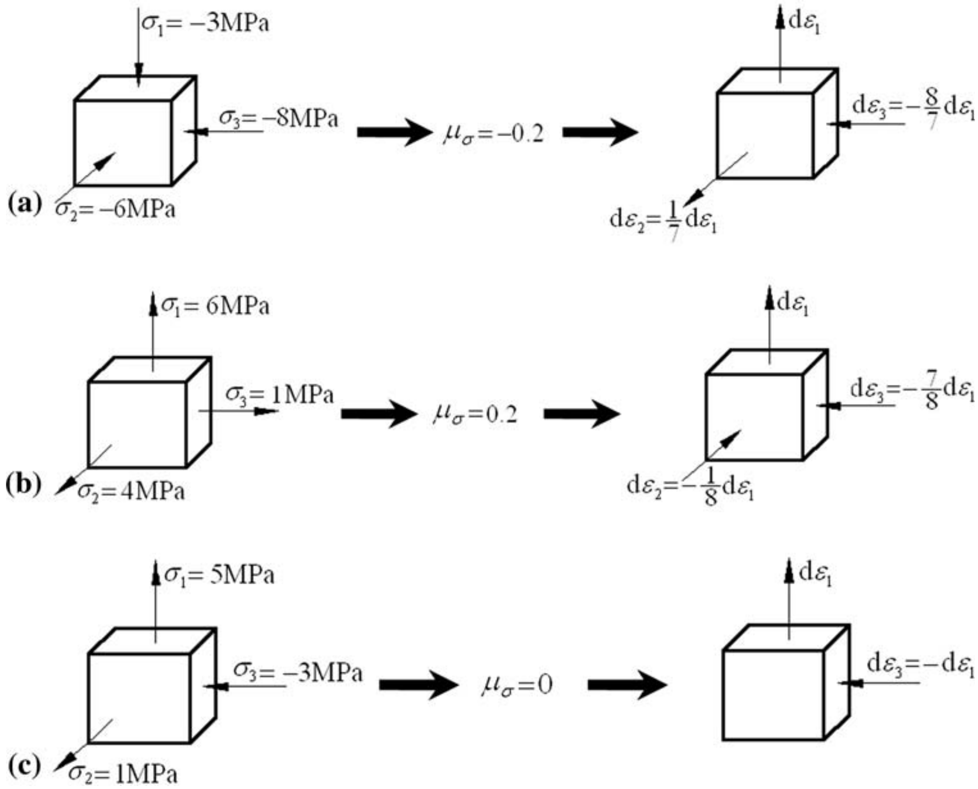

Fig. 4 Corresponding relationship between typical stress conditions and strain increments: (a) compression deformation, (b) elongation deformation, and (c) plane-strain deformation

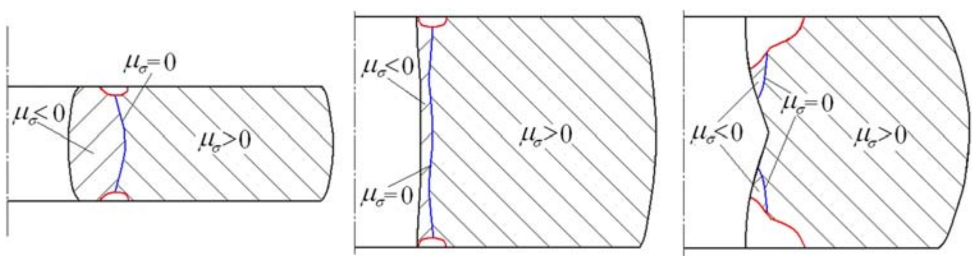

Fig. 5 Comparison of deformation division under different ratio of height to diameter: (a) λ = 0.5, (b) λ = 1, and (c) λ = 1.5

the inner side of the ring, and the Lode parameter value is negative on the left side of the line, namely, the elongation type of deformation happened in this area. But for the material on the right side of the line, compression type of deformation happened. With an increase of the height-diameter ratio λ, the elongation deformation area reduced while the compression deformation area expanded markedly, as shown in Fig. 5(b), and the location of the zero value line moved inward. When λ = 1.5, material in the middle part of the ring was in a compressive state which resulted in an uncontinuous zero value line, as can be seen in Fig. 5(c), the strain type of the small part of material inside the zero value line is elongation type of deformation.

Figure 6 shows the comparison of deformation division under different friction conditions. From Fig. 6(a), it can be seen that when the friction factor was low, all the material within the workpiece was in a plastic deformation state, and the strain type was homogeneous compression; with increasing friction factor, some difficult deformation zones appeared around the inner corners of the ring. Figure 6(b) shows the deformation division under a friction factor of 0.3 for which the Lode parameter value was negative in the material close to the inner wall, i.e. material strain type in this area was elongation type. When the friction factor was further increased, as shown in Fig. 6(c), the location of the zero value line of Lode parameter moved outwards in the radial direction, namely, elongation type deformation zone expanded remarkably, while the compressive deformation zone decreased.

4.2 Extrusion

The values of the Lode parameter were calculated and visualized for the extrusion process with different modular angles and the strain types in the plastic deformation zone were analyzed. Figure 7 shows the comparison of deformation division with different die angles, and the shadow zone is the plastic deformation area.

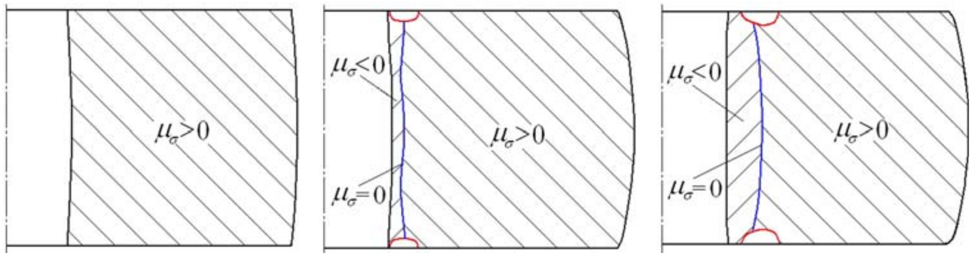

Fig. 6 Comparison of deformation division under different friction conditions: (a) m = 0.1, (b) m = 0.3, and (c) m = 0.6

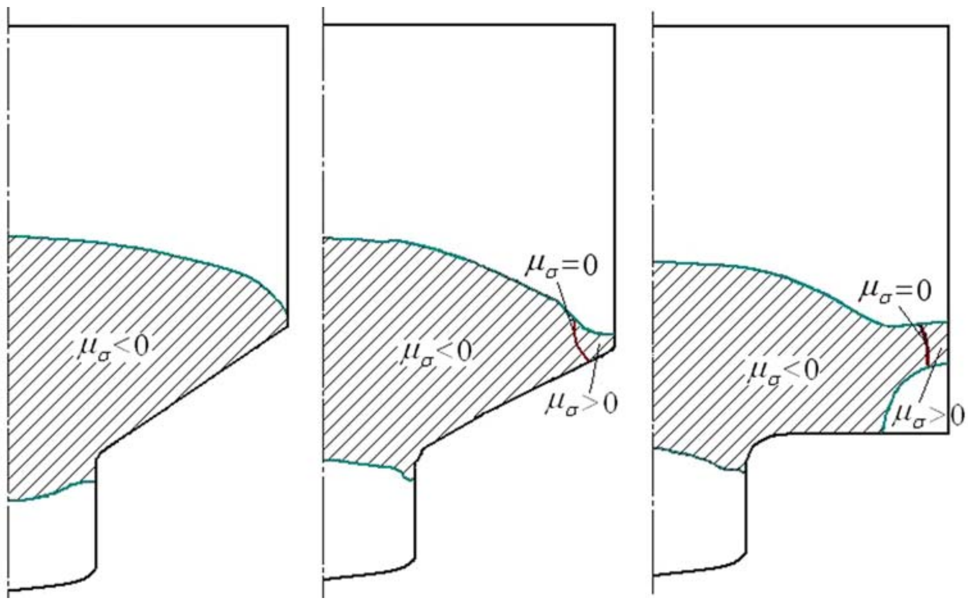

Fig. 7 Comparison of deformation division with different die angles: (a) 90°, (b) 120°, and (c) 180°

As can be seen from Fig. 7(a), the value of Lode parameter was negative in the plastic deformation area when the modular angle was 90°, which means the material was in the elongation type of strain state. With increasing modular angle, the material strain type changed markedly in the plastic deformation area and in the small area close to the bottom angle of the die cavity, the value of the Lode parameter was negative. In the plastic deformation area, strain type changes from one homogeneous elongation type to three coexisting forms: elongation type. In the plane-strain type and the compression type. Figure 7(c) shows the deformation division when the modular angle was increased to 180°, the plastic deformation area continuously decreased, and the outline also changed from a single arc to the shape of a near cosine curve, and a dead zone was found at the bottom angle area. Although the three kinds of strain types coexisted in the plastic deformation area, the area close to the sidewall with the compressive deformation increased. Compared with the other two situations, the metal flow behavior toward the die orifice was much more complicated, and the metal in the corner was not easy to be extruded.

Figure 8 shows the comparison of deformation division under different friction conditions. As can be seen from Fig. 8(a), during the direct extrusion process, the Lode parameter was negative in the majority region close to the die orifice, which means the material strain type was elongation. When the Lode parameter is zero, material strain type was plane-strain type. But in the plastic deformation area close to the sidewall, Lode parameter is positive, and the material strain type was compressive. But during the extrusion with active friction, as shown in Fig. 8(b), all the material in the plastic deformation zone was homogeneous elongation type. Therefore, compared with normal direct extrusion, material flow in the container was more homogeneous.

5. Conclusions

(1) In the plastic deformation situation, with increasing Lode parameter, the second deviatoric stress invariant increases and the deformation of work-hardening material increases.
(2) By using the quantitative relation between Lode parameter and the plastic deformation type, the relation between the type of plastic deformation and stress state can be obtained. By using post-processing of the FE code, the values of Lode parameter can be calculated and visualized, and the strain types within the workpiece can be obtained.

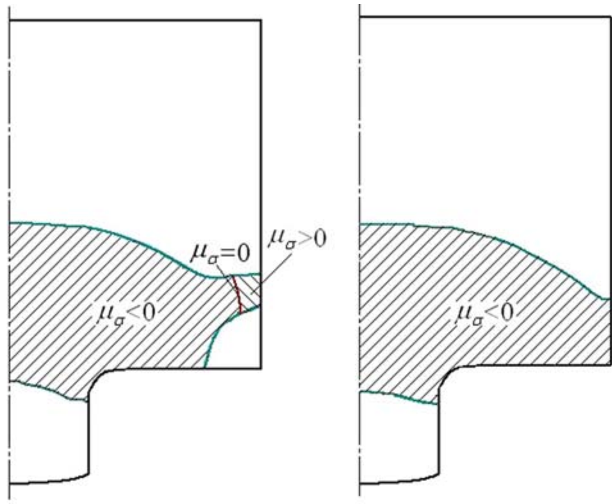

Fig. 8 Comparison of deformation division under different friction conditions. (a) Without active friction; (b) with active friction

(3) By comparing the deformation divisions in the ring compression process, with a further increase in the height-to-diameter ratio, the elongation type of deformation gradually reduces and the compression type of deformation area increases. With increases factor, the three strain types become coexistent from the homogeneous compression type in the workpiece.
(4) By comparing the deformation divisions in extrusion deformation process, the research indicates that as the mold angle increases, the three strain types become coexistent from homogeneous compression type in the deformable body. But during the extrusion with active friction, the strain type in the deformation area is elongation type. Therefore, compared with ordinary extrusion, metal flow in the container becomes much more homogenous.

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